The asymptotic mean value property for p-harmonic functions in the plane

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Classical results going back to Blaschke, Privalov and Zaremba say that if u is continuous in a domain $\Omega \subset \mathbb{R}^n$ and satisfies the asymptotic mean value property

$$u(x) = \fint_{B(x,r)} u(y)dy + o(r^2)$$

at each $x \in \Omega$ then u is harmonic in Ω. (The error term actually vanishes a posteriori by Gauss theorem). If $1 < p < \infty$, the following (nonlinear) asymptotic mean value property

$$u(x) = \frac{p-2}{p+n} \cdot \frac{1}{2}\Big(\sup_{B(x,r)} u + \inf_{B(x,r)} u \Big) + \frac{2+n}{p+n} \fint_{B(x,r)} u(y)dy + o(r^2) \quad (*)$$

is closely related to the p-laplacian. Manfredi, Parviainen and Rossi used the viscosity characterization of the p-laplacian and proved that continuous functions satisfying $(*)$ are p-harmonic. The converse, however, remains open for $n \geq 3$. More information is available when $n = 2$, due basically to the fact that the complex gradient of a p-harmonic function is a quasiregular mapping. Lindqvist and Manfredi used the hodographic method to show that p-harmonic functions in the plane satisfy $(*)$ with $n = 2$ and $1 < p < p_0 = 9.52....$ In the talk we will report joint work with A. Arroyo where we refine the method of Lindqvist and Manfredi and show that the same result holds in the whole range $1 < p < \infty$.

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