Regularity theory for weak solutions to degenerate Kolmogorov equations

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I present a survey of results concerning a family of degenerate second order linear Partial Differential Equations that arise in Stochastic theory and in its applications. This family includes, as a meaningful prototype, the following Kolmogorov operator in divergence form

$$\mathscr{L}u := \sum_{j,k=1}^{n} \partial_{x_j}\left(a_{jk}(x,y,t)\partial_{x_k}u\right) + \sum_{j=1}^{n} x_j \partial_{y_j} u - \partial_t u = \mathrm{div}_x\left(AD_x u\right) + \langle x, D_y u\rangle - \partial_t u,$$

where $(x,y,t) \in \mathbb{R}^{2n+1}$, and $A(x,y,t) := \big(a_{jk}(x,y,t)\big)_{j,k=1,\dots,n}$ is a symmetric, uniformly positive matrix, and the coefficients a_{jk}'s are bounded measurable functions.

In the first part of my talk, I describe the main known results of the regularity theory for classical and weak solutions. Then I will focus on the approach to the regularity theory based on the blow-up method. I first recall a local regularity resut for the obstacle problem, that has been obtained in a collaboration with M. Frentz, K. Nyström and A. Pascucci. Then I present a regularity result for classical solutions, recently obtained in collaboration with M. Eleuteri and B. Stroffolini. I will conclude with some partial results for the regularity of weak solutions, still obtained with M. Eleuteri and B. Stroffolini, and with some open problems.

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