The Liouville Theorem for Nonlocal Operator (and its relation to irrational numbers and subgroups of $\mathbb{R}^N$).

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The classical Liouville Theorem states that bounded harmonic functions are constant. In this talk we will revisit this result for a class of nonlocal operators. This class of operators naturally includes the fractional Laplacian, Relativistic Schrodinger operators, convolution operators, as well as discretizations of both local and nonlocal symmetric diffusion operators.

First, we will treat the one dimensional case. Here we give a precise classification for which the Liouville Theorem holds. The condition will be related to irrational numbers ([1]).

In the multi dimensional case such a characterization is also proved. This time it will be given in terms of additives subgroups of $\mathbb{R}^N$ ([1]).

This nonlocal result will allow us ([2]) to give a full characterization of the Liouville property for any linear operator (local + nonlocal, and not necessarily symmetric) with constant coefficients satisfying the maximum principle (see [3]).

References

[1] N. Alibaud, F. del Teso, J. Endal, and E. R. Jakobsen. *Characterization of nonlocal diffusion operators satisfying the Liouville theorem. Irrational numbers and subgroups of* $\mathbb{R}^d$. Preprint: arXiv:1807.01843.

[2] N. Alibaud, F. del Teso, J. Endal, and E. R. Jakobsen. *The Liouville theorem and linear operators satisfying the maximum principle. A complete characterization in the constant coefficient case.* Work in progress.

[3] P. Courrège. *Générateur infinitésimal d'un semi-groupe de convolution sur* R^n*, et formule de Lévy-Khinchine.* Bull. Sci. Math. (2), 88:3–30, 1964.

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