A random walk approach to the Robin boundary value problem

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We study the family of integral equations, called the Robin mean value equations (RMV), that are local averaged approximations to the Robin-Laplace boundary value problem (RL). When posed on $\mathcal{C}^{1,1}$-regular domains, we prove existence, uniqueness, equiboundedness and the comparison principle as well as the asymptotic Hölder equicontinuity (Lipschitz in the interior and α-Hölder at the boundary, for any $\alpha \in (0,1)$) for solutions to (RMV).

For any L^∞ right hand side f, we show that solutions to (RMV) converge uniformly, in the limit of the vanishing radius of averaging, to the unique $W^{2,p}$ solution, which coincides with the unique viscosity solution of (RL) for f continuous. We further prove the lower bound on solutions to (RMV), which is consistent with the optimal lower bound for solutions to (RL).

Proofs employ martingale techniques, where (RMV) is interpreted as the dynamic programming principle along a suitable discrete stochastic process, interpolating between the reflecting and the stopped-at-exit Brownian walks.

This is a joint work with Yuval Peres.

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