Streamlines of Infinity-Harmonic Functions

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My talk is based on a joint work with E. Lindgren (Uppsala).

The ∞-Laplace Equation

$$\Delta_\infty u \equiv \sum_{i,j=1}^{n} \frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}\frac{\partial^2 u}{\partial x_i \partial x_j} = 0$$

is the Euler-Lagrange Equation for the variational "integral"

$$\|\nabla u\|_{L^\infty(\Omega)} = \lim_{p\to\infty} \|\nabla u\|_{L^p(\Omega)}.$$

It was introduced by G. Aronsson in 1967 and has been rediscovered at least twice: for image processing (Caselles, Morel, and Sbert in 1998) and in stochastic game theory (Tug- of - War by Peres, Schramm, Sheffield, and Wilson in 2009).

The classical solutions of $\Delta_\infty u = 0$ have the property that *the speed* $|\nabla u(\overline{x}(t)|$ *is constant along a streamline* $\overline{x} = \overline{x}(t)$. Indeed,

$$\frac{d}{dt}|\nabla u(\overline{x}(t))|^2 = 2\,\Delta_\infty u(\overline{x}(t)),$$

if $d\overline{x}(t)/dt = \nabla u(\overline{x}(t))$. As we shall see, *for viscosity solutions this fundamental property fails* in the generic case. This is fatal for the regularity of the gradient. I shall treat only the two-dimensional case.

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